Constructing an Output Gap for Papua New Guinea

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Abstract

We construct a quarterly measure of the output gap for Papua New Guinea. To do so, we must first construct a quarterly measure of GDP. We do so by employing the procedure of [Chow, Lin, 1971], generalized by [Fernandez, 1981]. We use a number of criteria to ascertain which input indicator series to use in the Chow-Lin procedure. Having constructed a quarterly output series, we generate a variety of output gap measures using two statistical processes, the Hodrick-Prescott (HP) filter and the Beveridge-Nelson (BN) decomposition.

*The authors are available at nvellodi@bankpng.gov.pg and laba@bankpng.gov.pg respectively. We thank Jan Gottschalk of the IMF for his ongoing support, advice and guidance. The current paper is based largely on the structure of a workshop jointly run by the Pacific Financial Technical Assistance Centre (PFTAC) and the Bank of Papua New Guinea (BPNG), and hosted by the Reserve Bank of Fiji in Suva, August 2011, entitled "Macroeconomic Tools: Generating a Quarterly Indicator for Output". As such, we thank the participants for their suggestions. We also thank Jeffery Yabom, Acting Manager, Research Department BPNG, and Dr Gae Kauzi, Assistant Governor, Monetary and Economic Policy BPNG, for ongoing support for the project.

Contents

1 Introduction

In recent years, Papua New Guinea (PNG) has enjoyed a period of sustained economic growth. Rising commodity prices, and the commencement of major mining projects have been the main drivers of this growth. Gross Domestic Product (GDP) has grown by an average of roughly 7 percent annually over the last five years.[1] Concurrently, there is widespread consensus amongst fiscal and monetary authorities that significant domestic demand pressures have built up in this time. This phenomenon is of great interest to the Bank of Papua New Guinea (BPNG), as a build-up of domestic demand over and above the supply capacity of the economy may create inflationary pressures.

Since inflation is measured on a quarterly basis, analysis of this effect should be performed at the same frequency. Previous attempts have been made at creating a quarterly measure of output and of the output gap for PNG. [Lahari et al., 2009] constructs quarterly GDP series for a variety of South Pacific Island countries, including PNG, using the approach of [Chow, Lin, 1971]. This approach will lay the foundation for the first half of the present paper. [Sampson et al., 2006] use a variety of quarterly output measures, ranging from interpolating annual GDP data to using proxy variables for output such as non-mineral exports and the budget deficit, and then applying an HP filter to the resultant series. Whilst this approach is both innovative and novel in its choice and employment of appropriate proxy variables for output, it does not combine the information contained in both the quarterly proxies and the annual GDP series, thereby loosing valuable information. In this paper, we will address this issue by adopting a systematic and rigorous method for ascertaining which quarterly variables may proxy GDP well, and then combining these variables with the official annual GDP data in the manner of [Chow, Lin, 1971] and [Fernandez, 1981].[2] The Chow-Lin procedure is a method of temporal disaggregation that entails significant advantages over standard interpolation. Most importantly, it combines relevant high-frequency series with the low-frequency series, thus incorporating important quarterly information whilst maintaining the overall profile of the high-frequency series.

This paper is the first in a two-part analysis. This installment is devoted solely to constructing a quarterly measure of the output gap. As such, there is no

[1] Based on Treasury and Bank of Papua New Guinea estimates.

[2] Henceforth, we will refer to these approaches as the Chow-Lin and Fernandez procedures respectively.

estimation of a structural model. For such results, we refer you to the second installment, which will use this measure to estimate a Phillips Curve for PNG. This paper is structured as follows. We begin by creating a quarterly series for GDP. This involves choosing a set of potential quarterly indicator series that are seen to trend with GDP, both by using principal component analysis (PCA), as well as simply eye-balling the variables. Having "pre-filtered"the data, we employ both the Chow-Lin procedure and a slight variant of this procedure, as described in [Fernandez, 1981]. Having created the quarterly output series, we construct various measures of the output gap. This step involves using two different statistical methods for detrending output into potential and cyclical components, each method entailing a different economic interpretation of the output gap. We conclude by briefly describing the economic narrative embodied in our final measure of the output gap, and its relevance for inflation in PNG.

2 Creating the Quarterly Output Series

2.1 Real versus Nominal Output

Before we proceed, it is worth clarifying an important point regarding whether we are considering real or nominal output. Insofar as the final objective of this paper is to construct an output gap, using real output is preferred. With this in mind, there are two potential approaches we could adopt. First, we could consider annual nominal inflation, use nominal quarterly series, create a quarterly nominal output series and then deflate this series using a quarterly deflator. Such a quarterly deflator could be either CPI, or preferably, a quarterly GDP deflator constructed by applying the Chow-Lin procedure to the annual GDP deflator, using headline CPI as the sole quarterly series. Second, we could take annual real output, use real quarterly series and create a quarterly real output series directly.[3] In this paper, we opt for the later, mainly because a number of potentially valuable quarterly series, such as employment series, are available only in real terms. As such, all potential quarterly series are expressed in constant prices.

[3] The former approach was adopted by [Lahari et al., 2009].

2.2 Pre-filtering

Our approach to choosing the best possible indicators involves narrowing down from a large range of potential indicators to just a handful. First, we compile a large database of all potential indicators of real GDP, drawing from monetary, fiscal, balance of payments and production data. For details on these data, refer to Appendix A. Next, we perform a simple eye-balling exercise, annualizing these series and plotting the results against annual output. At this stage, we can immediately rule out many series that clearly do not trend with annual output, since it is highly unlikely that they will represent quarterly output in this case. We then turn to a more rigorous means of ruling out possible indicators by performing a principal component analysis between the potential quarterly indicators.

The results from the eye-balling exercise are shown in Figures 1 to 4. The quarterly indicators have been both annualized and transformed into index form, thus making both the trend and scale of each series comparable to the annual output series. From the graphs alone, we can immediately rule out a number of variables. In Figure 1, it is only the variables "Currency outside depository corporations"that follows the trend and growth rate of output; the other variables grow far faster than output. In Figure 2, both "Personal tax"and "Recurrent Expenditures"have roughly the correct trend and growth rates. A similar analysis for the other charts yields the first-round filtering short-list of seven variables, as described in Table 1.

The final stage of pre-filtering involves a technique known as principal component analysis (PCA). For a given set of variables, PCA finds combinations of the individual series that account for most of the variance in the data.[4] If we have a number of variables that are assumed to trend with the same overall variable (in this case output), redundancy occurs if variables are correlated. To this end, PCA is often used to deal with multicollinearity in multiple regression analysis. The PCA generates two important sets of outputs. The *eigenvectors* are combinations of the variables that effectively tell us how significant each of them are in explaining the variance in the data, whilst the *eigenvalues* effectively tell us which of the eigenvector combinations best account for the overall variance. As such, we are mainly interested in the eigenvector with the largest associated eigenvalue, and the highest weighted variables within this eigenvec-

[4] For the sake of brevity, we will refrain from providing a lengthy discussion of PCA.

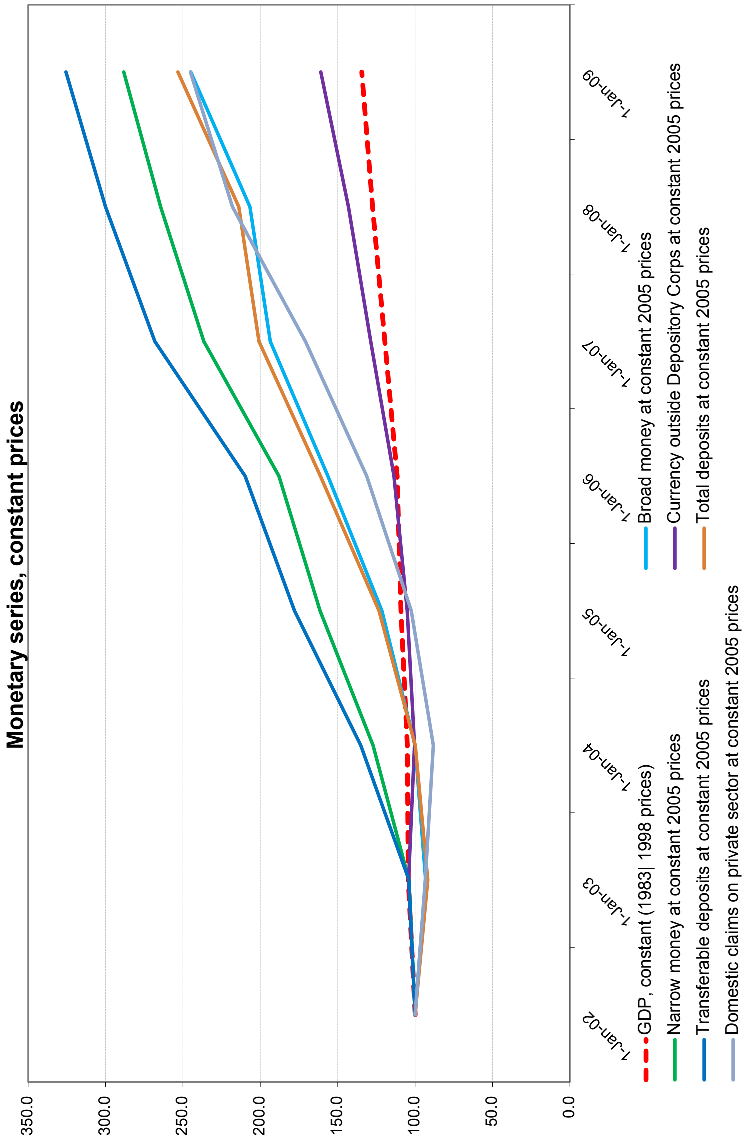

Figure 1: Monetary Indicators

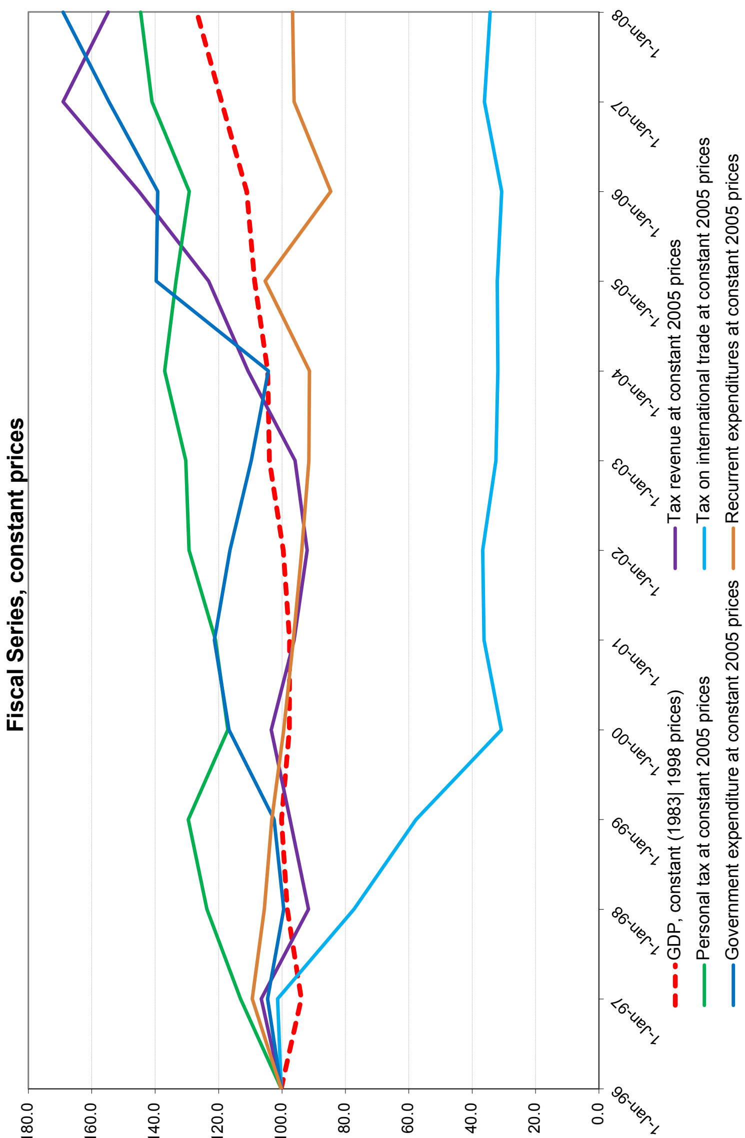

Figure 2: Fiscal Indicators

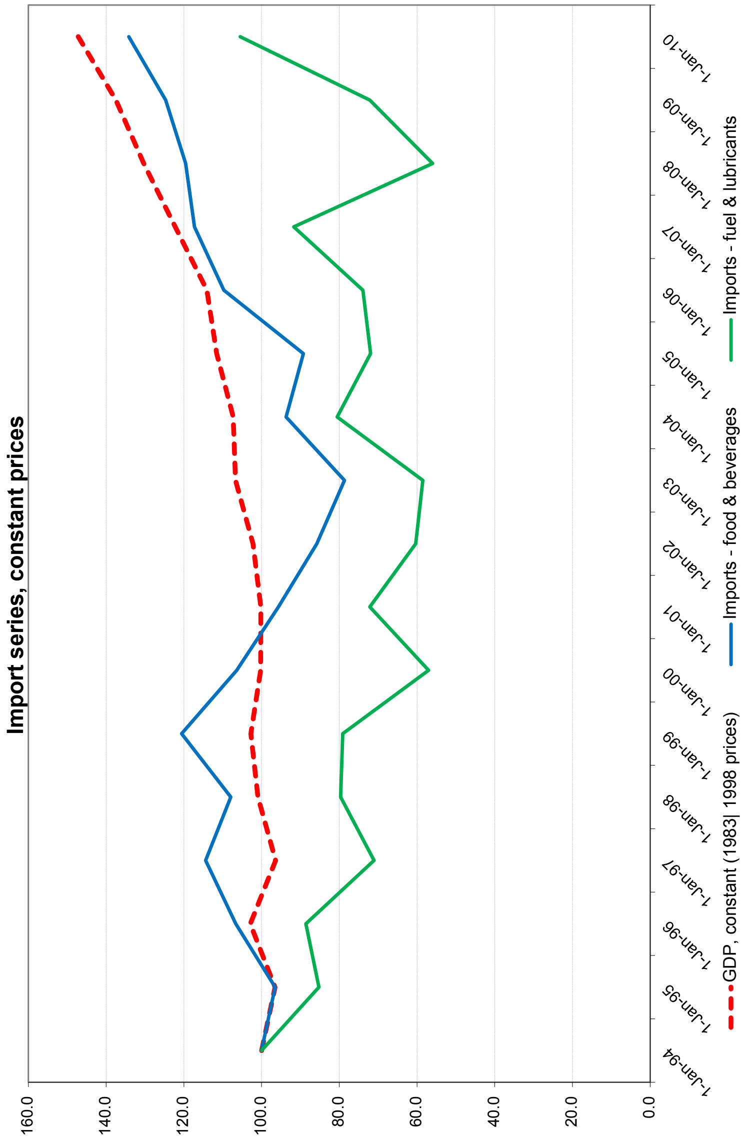

Figure 3: Import Indicators

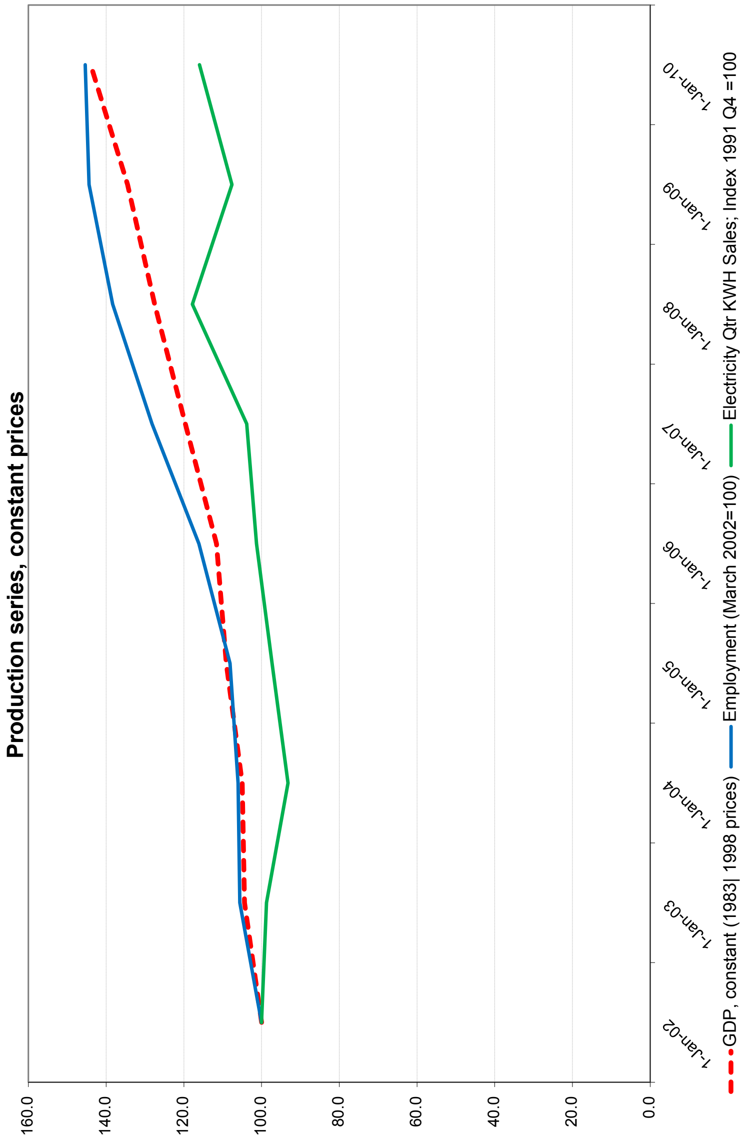

Figure 4: Production Indicators

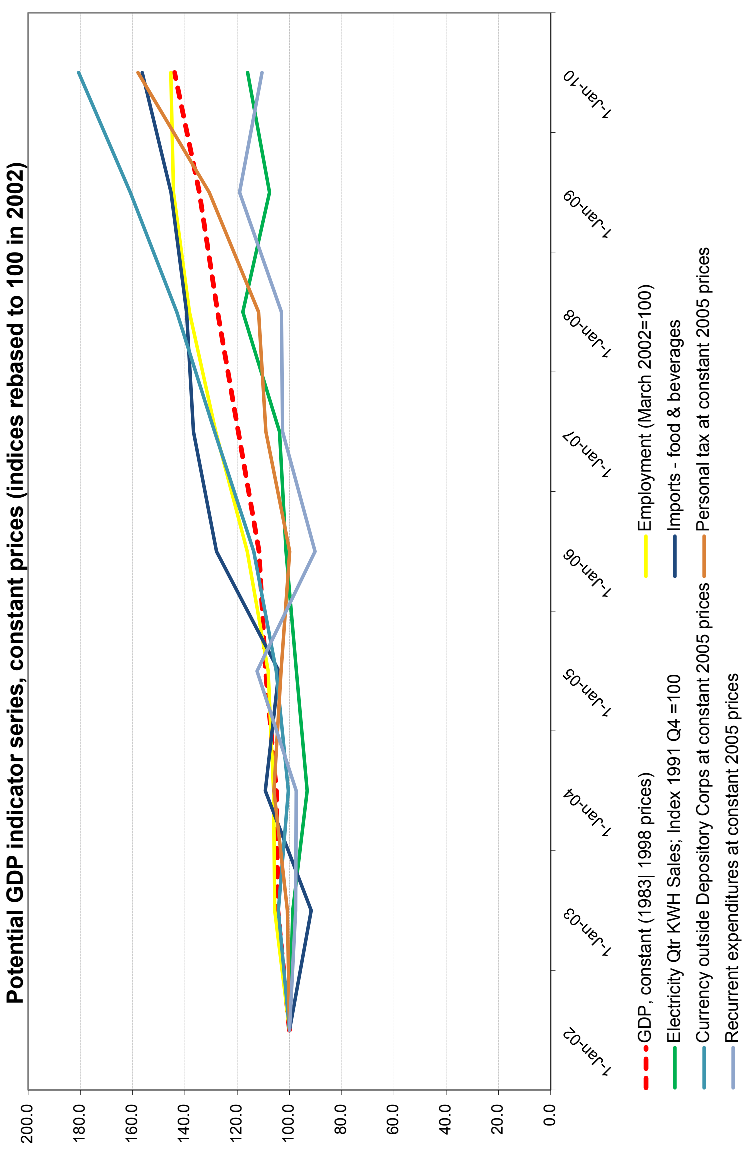

Figure 5: Potential Indicators

Table 1: **Results of eye-balling exercise**

Variable	Keep or discard?
Broad money	Discard
Narrow money	Discard
Currency outside depository corporations	Keep
Transferable deposits	Discard
Total deposits	Discard
Domestic claims on private sector	Discard
Personal Tax	Keep
Tax revenue	Discard
Tax on international trade	Discard
Government expenditure	Discard
Recurrent expenditure	Keep
Imports - food and beverages	Keep
Imports - fuel and lubricants	Discard
Employment - non-mineral	Keep
Employment - mineral	Keep
Electricity sales, kwh	Keep

tor. The results from the PCA performed on the seven short-listed variables are summarized in Table 2.

It is clear that the first eigenvector explains the highest proportion of the overall variance in the data, and that within this eigenvector, the "recurrent expenditures" and "food imports" series capture the least part of the variance, i.e. are the weakest. The remaining variables account for relatively similar proportions of the overall variance. Since this method is employed here only as an indication, it would be unwise to exclude too many variables based on its findings. Hence, we eliminate only the "recurrent expenditures" and "food imports" variables at this stage.

At the end of the pre-filtering process then, we are left with the final choice of indicator variables as described in Table 3.

2.3 The Chow-Lin Procedure

Having selected a shortlist of five indicators, this section proceeds to discuss the Chow-Lin procedure and the Eviews implementation used in this paper.[5] As intimated in the introduction, the procedure is a method of temporal disag-

[5] The Eviews source code listing for the procedure is contained in Appendix D.

Principal Components Analysis
Date: 06/21/12 Time: 15:45
Sample (adjusted): 1996Q1 2010Q2
Included observations: 58 after adjustments
Balanced sample (listwise missing value deletion)
Computed using: Ordinary correlations
Extracting 7 of 7 possible components

Eigenvalues: (Sum = 7, Average = 1)

Number	Value	Difference	Proportion	Cumulative Value	Cumulative Proportion
1	4.709619	3.527097	0.6728	4.709619	0.6728
2	1.182522	0.711550	0.1689	5.892141	0.8417
3	0.470972	0.081487	0.0673	6.363113	0.9090
4	0.389485	0.224953	0.0556	6.752598	0.9647
5	0.164532	0.109847	0.0235	6.917130	0.9882
6	0.054686	0.026501	0.0078	6.971816	0.9960
7	0.028184	---	0.0040	7.000000	1.0000

Eigenvectors (loadings):

Variable	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7
TAX_PER_TC	0.384705	-0.238966	0.603871	-0.081533	0.600196	0.050511	0.246583
KWH_TC	0.382308	-0.116025	-0.605452	0.496768	0.367382	-0.285621	0.102364
IMP_FOOD_TC	0.321039	0.468998	-0.370300	-0.679904	0.227568	0.152450	0.050541
EXP_CURR_TC	0.161058	0.793715	0.279309	0.493437	-0.036640	0.133293	0.058942
EMP_NONMIN_TC	0.437774	-0.142851	-0.001679	-0.041035	-0.608305	0.024796	0.644679
EMP_MIN_TC	0.434366	-0.237237	-0.058739	0.139282	-0.145535	0.677361	-0.502191
CURRENCY_TC	0.443301	0.052948	0.224006	-0.140501	-0.245866	-0.644538	-0.504859

Ordinary correlations:

	TAX_PER_TC	KWH_TC	IMP_FOOD...	EXP_CUR...	EMP_NON...	EMP_MIN_TC	CURRENC...
TAX_PER_TC	1.000000						
KWH_TC	0.573691	1.000000					
IMP_FOOD_TC	0.388653	0.499253	1.000000				
EXP_CURR_TC	0.128446	0.192791	0.504154	1.000000			
EMP_NONMIN_TC	0.778837	0.765065	0.572186	0.194797	1.000000		
EMP_MIN_TC	0.816910	0.837509	0.498020	0.130831	0.939810	1.000000	
CURRENCY_TC	0.826820	0.693600	0.682465	0.384361	0.921662	0.867346	1.000000

Table 2: Principal Components Analysis

Table 3: **Final Choice of Indicators**

Currency outside depository corporations
Personal Tax
Employment - non-mineral
Employment - mineral
Electricity sales, kwh

gregation that transforms an annual series into a quarterly series by combining other quarterly series (herein refered to as *indicator series* or simply *indicators*) that relate to the underlying annual series.[6] It offers many advantages to basic interpolation. Firstly, the annual GDP series may not entirely accurate, and hence may not fully represent actual output. This is particularly relevant in the case of PNG, where official GDP data as compiled by the National Statistics Office has been unavailable since 2006, and hence GDP is based on Ministry of Treasury estimates.[7] In this case, it is clearly preferable to use a range of indicators that may provide additional relevant information above and beyond the annual series. Secondly, interpolation, by definition, assumes a quarterly profile for the annual series. The Chow-Lin procedure uses the information contained in the quarterly indicators to generate a non-trivial quarterly profile.

There are other practical benefits. Currently, the primary statistical software used at the Bank of Papua New Guinea (BPNG) is Eviews, version 7. Since the procedure is computationally relatively simple, writing an Eviews program to execute the procedure is a relatively straightforward affair.[8]

2.3.1 Theory

We will keep exposition of the technical aspects of the procedure to a minimum.[9] Let y_a be the annual series in question, y the unknown quarterly series and X the matrix of quarterly indicators. Thus, in our case, y_a is annual output, y is quarterly output and X is a 5-column matrix comprising of our five chosen indicators.

We assume a linear relationship holds between y and X of the form:

$$y = X\beta + \epsilon$$

where $E(\epsilon) = 0$ and $E(\epsilon\epsilon') = \Omega$. Let C be an aggregation matrix that premultiplies with y to give y_a:

[6] The procedure works from any low frequency to high frequency conversion, but we are specifically concerned with annual to quarterly conversion.

[7] See the Budget 2012, Vol 1.

[8] To the authors' best knowledge, there is no in-built functionality in Eviews 7 or previous releases that performs the Chow-Lin procedure, hence the need to write the routine from scratch. Such functionality exists for a range of other software, including RATS, STATA, GAUSS, OxMetrics and MATLAB.

[9] For further details, see [Chow, Lin, 1971] and [Frain, 2004].

$$C = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & 1 & 1 \end{pmatrix} \tag{1}$$

Then $y_a = Cy$ and $X_a = CX$, where X_a is the matrix of annualized quarterly indicators.[10]

The aim is to find the best linear unbiased estimator for y in terms of C and Ω. This is given by the expression:

$$\hat{y} = X\hat{\beta} + \Omega C'(C\Omega C')^{-1}\hat{u}_a$$

where

$$\hat{\beta}_a = \left[X_a'(C\Omega C')^{-1}X_a\right]^{-1} X_a'(C\Omega C')^{-1}y_a \tag{3}$$

$$\hat{u}_a = y_a - X\left[X_a'(C\Omega C')^{-1}X_a\right]^{-1} X_a'(C\Omega C')^{-1}y_a \tag{4}$$

As [Frain, 2004] notes, the expression for $\hat{y}$ can be given an intuitive interpretation; the final quarterly series is the sum of two components:

- $X\hat{\beta}$ is the estimated regression coefficients applied to the quarterly indicators.
- $\Omega C'(C\Omega C')^{-1}\hat{u}_a$ is the residual in the annual regression distributed over the quarters.[11]

Aside from choosing the indicators, the implementation of this procedure involves one major difficulty, namely that the covariance matrix Ω is unknown. Hence,

[10] C takes the annual figure to be the arithmetic mean over the four quarters. The aggregation matrix C is given for interpolation of a flow variable. If a stock variable is used, then we require

$$C = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

[11] This ensures that the final quarterly series sums to the known annual series.

we must assume a certain structure for it. [Chow, Lin, 1971] assume that these innovations follow a stationary AR(1) process, and are homoskedastic. These assumptions are most succinctly described in the state-space representation:[12]

$$\begin{aligned} y_t &= x_t\beta + u_t \quad (5) \\ u_t &= \rho u_{t-1} + \epsilon_t \quad (6) \end{aligned}$$

where $|\rho| < 1$, $u_1\ N(0, \sigma^2/(1-\rho^2))$ and $\epsilon_t\ iid, N(0, \sigma^2)$. [Fernandez, 1981] considers the case of non-stationary innovations, i.e. $\rho = 1$, whilst [Litterman, 1983] models the first difference of the innovations as an AR(1), thus rendering the state-space representation:

$$\begin{aligned} y_t &= x_t\beta + u_t \quad (7) \\ \Delta u_t &= \rho\Delta u_{t-1} + \epsilon_t \quad (8) \end{aligned}$$

In our implementation of the procedure, we allow for innovations in the errors ϵ that are are heteroskedastic and follow a "pseudo-AR(1)"process.[13] For further details, see Appendix B.

2.3.2 Application

Having chosen our quarterly series in Section 2.2, we can now run the Chow-Lin procedure, using various combinations of the quarterly series as inputs. Owing to the vast quantity of possible combinations, we restrict our output to a few hand-selected combinations.[14] Chart 6 shows three combinations of quarterly input series. The series labeled "gdp_cnst_qrtly"is simply annual GDP uniform distributed over the four quarters in each year.

It is important to observe that the plots for all three quarterly output series thread through gdp_cnst_qrtly, demonstrating that the procedure is distributive. The series labelled "tax_per, kwh"took personal income tax and electricity pro-

[12] See [Proietti, 2004].

[13] In the actual implementation, we make the simplifying assumption that the innovations are homoskedastic. This assumption is easily altered in the code, but was seen to make little impact on the overall output of the procedure.

[14] For six quarterly indicators, there are $\sum_{i=1}^{6} \binom{6}{i} = 63$ combinations as inputs.

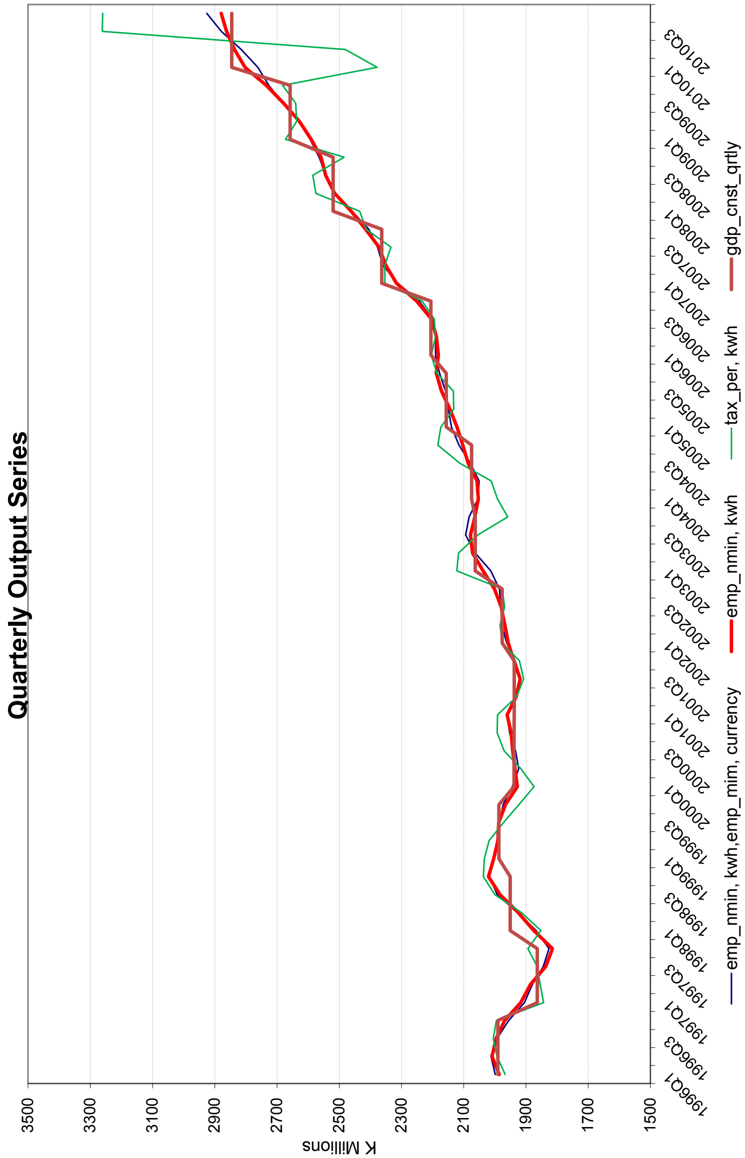

Figure 6: Quarterly Output Series, for different combinations of quarterly input series

duction as inputs, and is a good example of one potential failing of the Chow-Lin procedure, namely that there is nothing prohibiting the endogenously determined value of the autocorrelation parameter ρ generating highly volatile quarterly output series. This problem is addressed in the routine listed. See Appendix B for further details. It is hard to discriminate visually between the two remaining quarterly output series. We shall proceed with both of them, and use the output gap measures to further discriminate.

2.3.3 Extensions

We began this section by outlining some of the benefits entailed in adopting the Chow-Lin procedure. However, there are some drawbacks involved, which have been addressed through a number of extensions to the Chow-Lin procedure. Firstly, as outlined previously, the extensions proposed in [Fernandez, 1981] and [Litterman, 1983] model the autocorrelation in the innovations differently. [Litterman, 1983] argues that the approach of [Fernandez, 1981] performs significantly better than the Chow-Lin procedure. He also proposes his approach is an improvement on [Fernandez, 1981]. We wrote code to perform the Fernandez procedure, and as such, unless otherwise stated, will proceed using the Fernandez procedure.[15] Other extensions were proposed by [Hendry, Mizon, 1978] and [Harvey, Chung, 2000], the former generalizing the Litterman approach further, and the latter providing a multivariate approach to disaggregation.

3 Constructing the Output Gap

As intimated in the introduction, the ultimate purpose of this paper is to construct a quarterly measure of demand-pull inflationary pressures for PNG. The process through which such inflationary pressures manifest can be characterized as follows: The supply capacity of the economy is temporarily fixed, as factors of production are finite (there are a limited number of workers, and capital equipment). Hence, an increase in demand will lead to short-run upward price pressures as firms adjust prices upwards to meet this additional demand. Thus,

[15] We generated output measures using both procedures, and found little difference between the results, when the autocorrelation parameter in the Chow-Lin procedure was calculated using the approach outlined in Appendix B. The Fernandez approach was favoured as per the reasoning of [Litterman, 1983]. The Litterman procedure presented severe difficulties when deriving a maximum likelihood function to determine the auto-correlation parameter for the first-difference innovation terms, and as such was disregarded.

we require a measure of the *excess* of aggregate demand over the aggregate supply capacity of the economy. As such, a quarterly output series is necessary but insufficient, since quarterly output cannot capture this mismatch itself. We must instead consider the *output gap*. The output gap measures the difference between actual output and potential output, where potential output may be defined as the level of output that can be sustained in the long-run.[16]

Indeed, how we define potential output, and hence the output gap, plays a crucial role in how these variables should be estimated. Roughly speaking, we may think of three approaches to defining potential output. The first is statistical, i.e. potential output is simply the statistical trend component of actual output. Methods of estimating potential output consistent with this approach would be univariate filters such as the Hodrick-Prescott (HP) Filter and Beveridge-Nelson (BN) decomposition, as well as state-space models, such as the Unobserved Components (UC) method and generalized Kalman Filter approaches.[17] The second is structural, i.e. potential output as representing the supply capacity and structure of production within the economy.[18] Estimation methods for this approach include the production function approach or estimating Okun's Law.[19] The third is a combination of the first two, i.e. using a structural approach combined with statistical filtering. Structural Vector Autoregression (SVAR) analysis, as well as multivariate filtering (Kalman, HP filters) approaches fit into this category.

In this paper, we will employ a simple univariate filter, applied to the quarterly output series rendered in the previous section. As such, our approach best fits into the third category. After all, the Chow-Lin procedure imposes a linear relationship between output and relevant indicator variables, thus embodying an economic relationship, whilst the HP filter falls squarely into the statistical category. Indeed, in the case where the chosen quarterly series are employment and electricity production, the Chow-Lin procedure can be interpreted as estimating a production function.[20] We find this approach to be both practical and rigor-

[16] This definition is far from universal. There is a substantial literature devoted to the definition and estimation of potential output. We shall not attempt to reflect its manifold hypotheses in the current paper.

[17] See [Hodrick, Prescott, 1980], [Beveridge, Nelson, 1981] and [Watson., 1986] for the primary expositions of the HP filter, BN decomposition and UC methods respectively.

[18] See [ECB, 2000].

[19] [Proietti et al., 2007] estimate the output gap for the Euro area using a production function approach, whilst [Attfield, Silverstone., 1998] adopt an Okun's Law approach, combined with cointegration analysis.

[20] It is clear that the non-mineral employment series represents labor as a factor of production, L, whilst many studies have adopted electricity consumption as a proxy for the capital stock, K.

ous. Estimating structural relationships requires high quality data, the absence of which may yield weak or non-sensical results. On the other hand, purely statistical approaches often lack a certain economic intuition, and the results can be hard to provide an economic interpretation of.

3.1 Univariate Filtering

We will apply two univariate filters, namely the HP filters and BN decompositions. For the HP filter, we choose the parameter λ according to the methodology outlined in [Marcet, Ravn., 2003]. We run the BN decomposition on a variety of ARMA specifications for the autocorrelation structure in the deterministic trend component. See Appendix C for further details. Figures 7 to 10 show the results.[21]

It may seem disconcerting that the output gap measures derived through the HP filters and BN decompositions differ to such a high degree. Indeed, they appear to exhibit almost no correlation whatsoever. This is readily explained by observing the trend output components estimate by each method. The HP filter clearly models potential output as adjusting at a far slower rate than the BN decomposition, which models potential output as accounting for the major part of output volatility. Which one we choose depends on our view on potential output in PNG.[22] We feel that the HP filter estimates of trend output are a more realistic representation of the evolution of potential output in PNG, and hence will adopt this as our final choice of univariate filter. Furthermore, there is little difference between the gap measures including and excluding "currency in circulation"as a quarterly input variable. As such, we will exclude it from out input variables set, as an output series based on purely production variables (non-mineral employment and electricity production) embodies a more elegant economic interpretation.

See, for instance, [Vial., 2006] and [Levinsohn, Petrin., 2003]. The annual regression function in the Chow-Lin procedure thus becomes:

$$y_{4t} = \beta_0 + \beta_1 L_{4t} + \beta_2 K_{4t}$$

[21] For the HP filters, we set $\lambda = 1600$, and for the BN decompositions, we use an ARMA(2,1) structure for the autocovariance structure for Δy_t.

[22] [Morley et al., 2002] discuss the divergence between BN and UC decompositions of output, whilst giving intuitive explanations why how this divergence relates to each filter's definition of potential output.

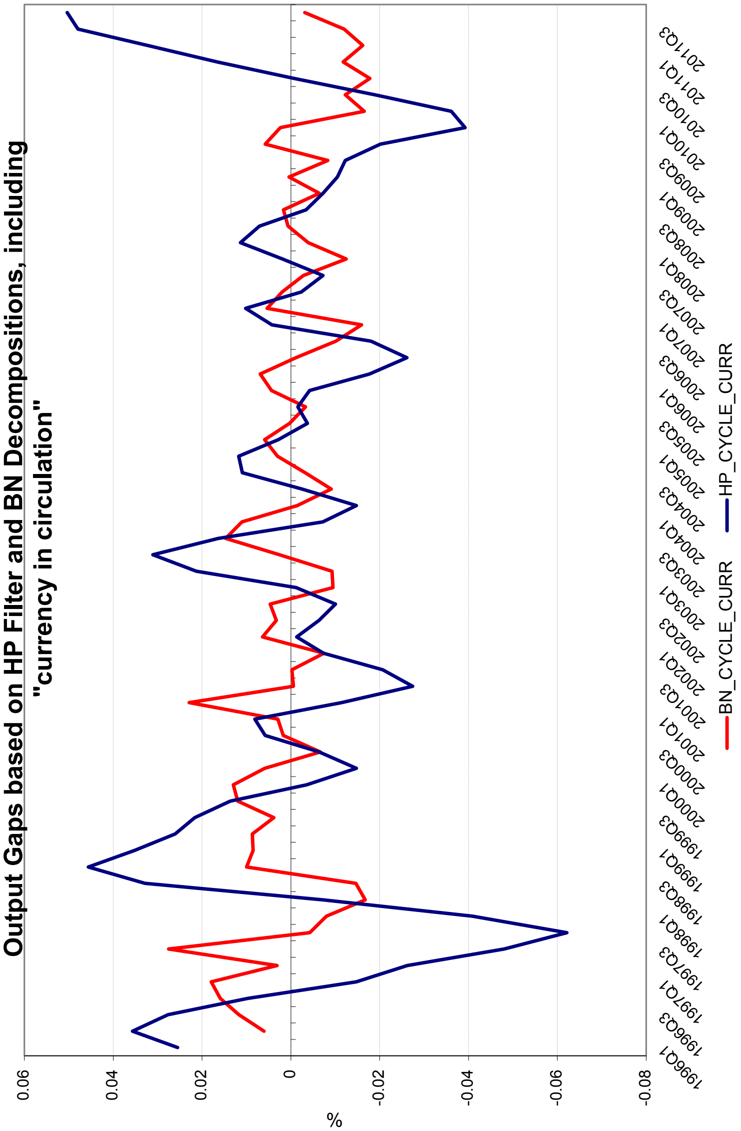

Figure 7: Output gap measures, using non-mineral employment, electricity production and currency in circulation as quarterly input series

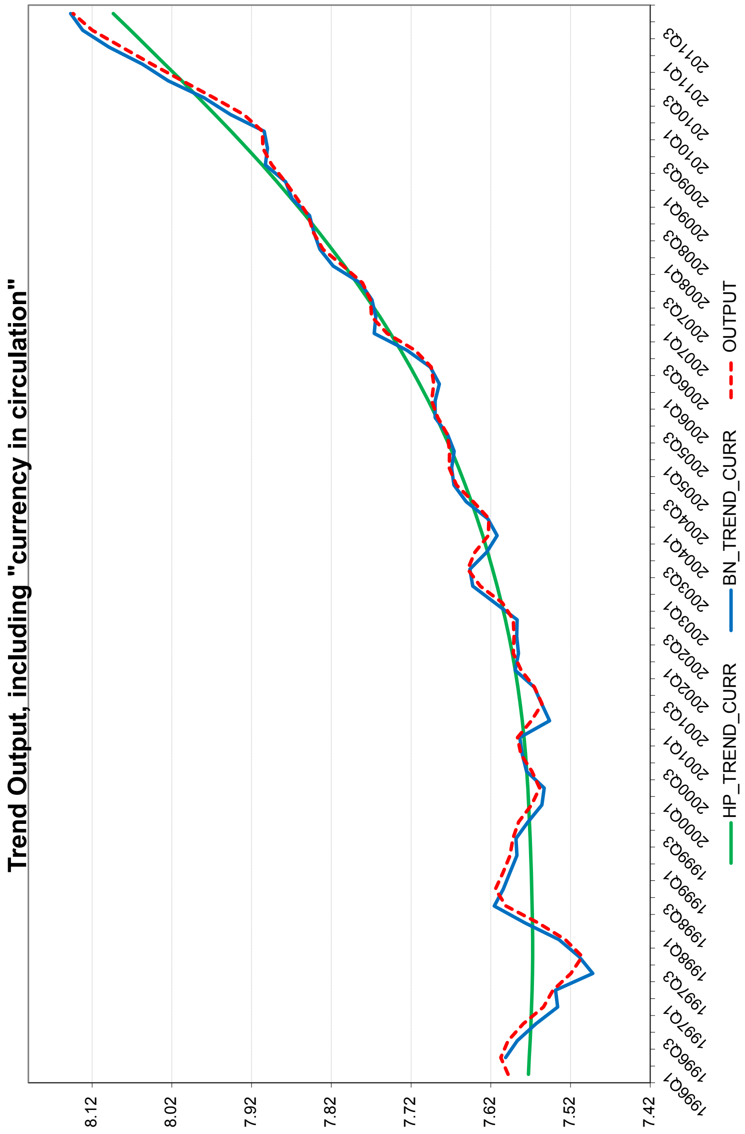

Figure 8: Trend output measures, using non-mineral employment, electricity production and currency in ciruclation as quarterly input series

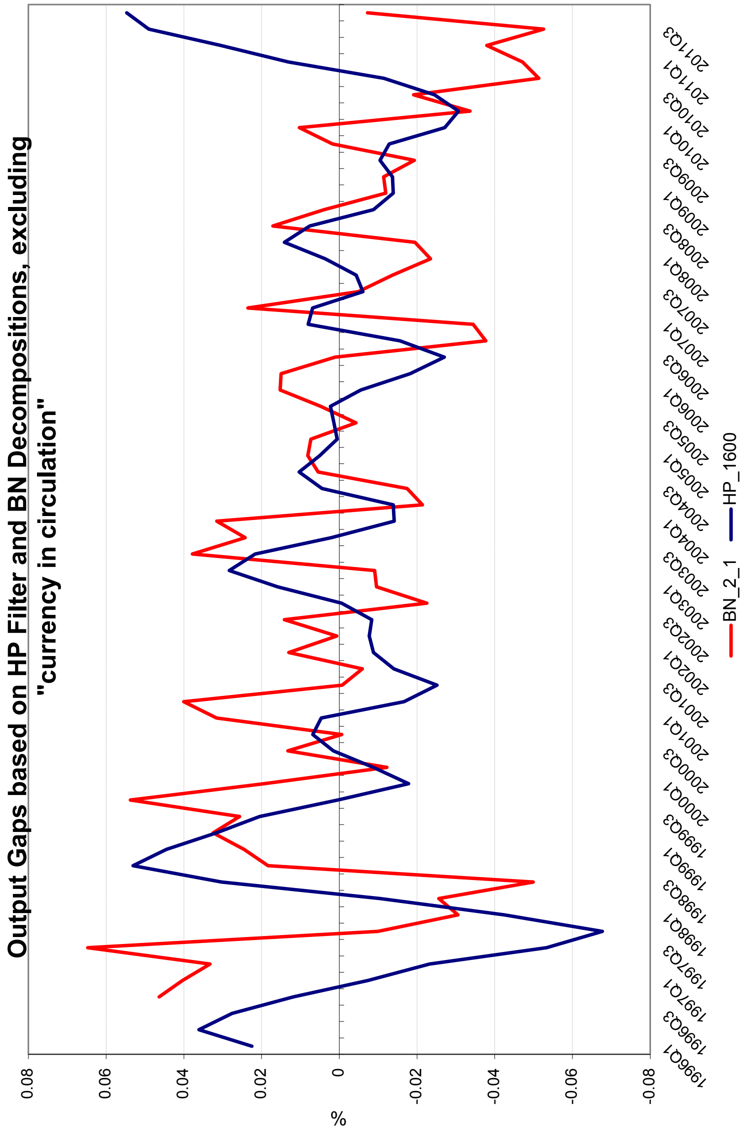

Figure 9: Output gap measures, using non-mineral employment and electricity production as quarterly input series

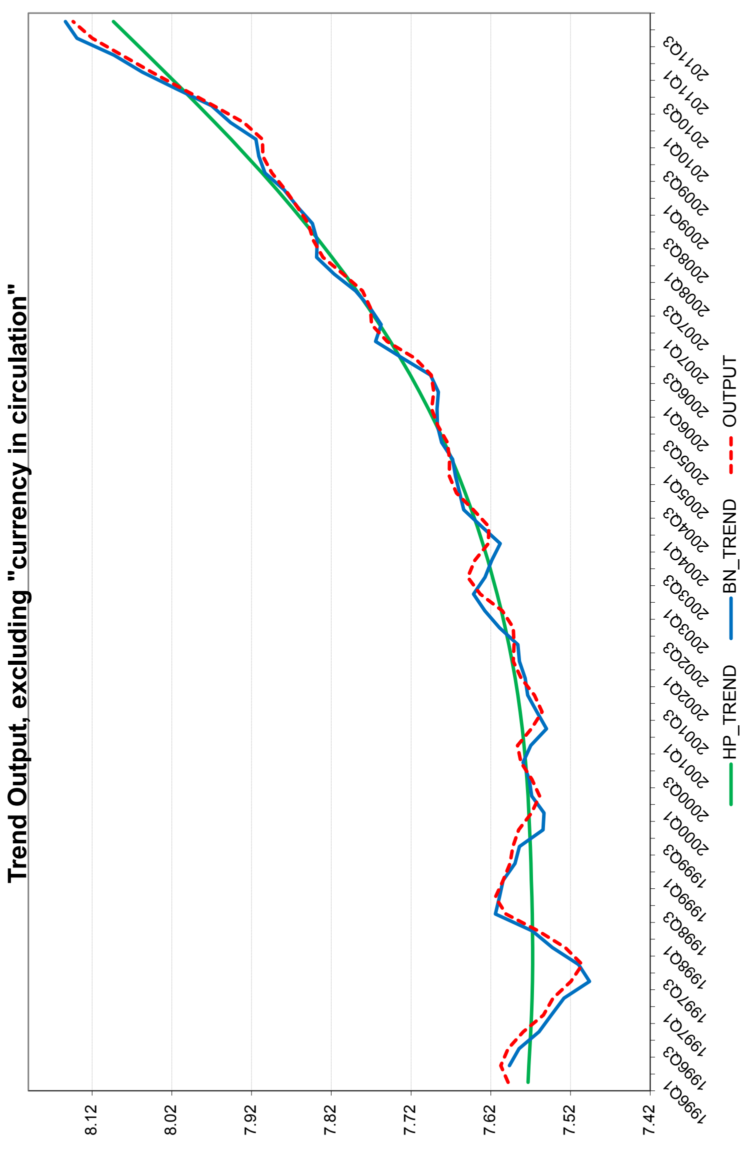

Figure 10: Trend output measures, using non-mineral employment and electricity production as quarterly input series

3.2 Interpretation

Figure 11 shows the final choice of output gap measure, using non-mineral employment and electricity production as quarterly indicators, and the univariate HP filter with $\lambda = 1600$. At this point, it is instructive to try and interpret the economic narrative told by this measure, i.e. what does it tell us about demand conditions in PNG over the sample period, and does this story match up with the stylized facts of actual experience? Of particular note is the year 2011, where the output gap is at a historical high. This is in keeping with the notion that PNG has been experiencing a boom, fulled by the major PNGLNG natural gas project and associated activities.[23] There is also some evidence to support the height of the commodity price boom that occurred in the immediate run-up to the global financial crisis in 2008Q3. However, there are some areas of concern. Notably, the period around 1997Q3 - 1998Q1, in which the output gap turns highly negative, implying a deep recession was experienced in PNG at this time, followed by a short period of strong growth from 1998Q2 - 1999Q3. To the authors' best knowledge, this story does not match the reality of PNG's experience around this period in history, and hence we must see this as a failing of our choice of output gap in accurately reflecting historical demand conditions. Of course, there are several alternative measures of the output gap that we could consider, two of which stand out. First, we could consider using annual non-mineral output rather than total output. This approach assumes that mining output has little or no impact of domestic demand conditions, as mining companies often provide their own factors of production. Secondly, one could disregard entirely the annual GDP series and simply form a weighted average of the underlying indicators to create a quarterly output measure, using the weights endogenously determined in the Chow-Lin or Fernandez procedures.[24] The output gap is again constructed using the univariate HP filter. We refer to these output gaps as the *non-mineral* and *composite output gaps* respectively. For the sake of brevity, we omit detailed expositions of these alternative approaches.

[23] The project has large spillover effects in manufacturing, construction and transport. See the March 2012 Quarterly Economic Bulletin.

[24] More specifically, the Chow-Lin procedure assigns GLS estimator values for the quarterly weights, given by equation (3). The weighted average of the indicator series would then become

$$\frac{\hat{\beta}}{||\beta||} X = \frac{1}{||\beta||} \sum_{i=1}^{p} \hat{\beta}_i X_i$$

where p is the number of quarterly indicators.

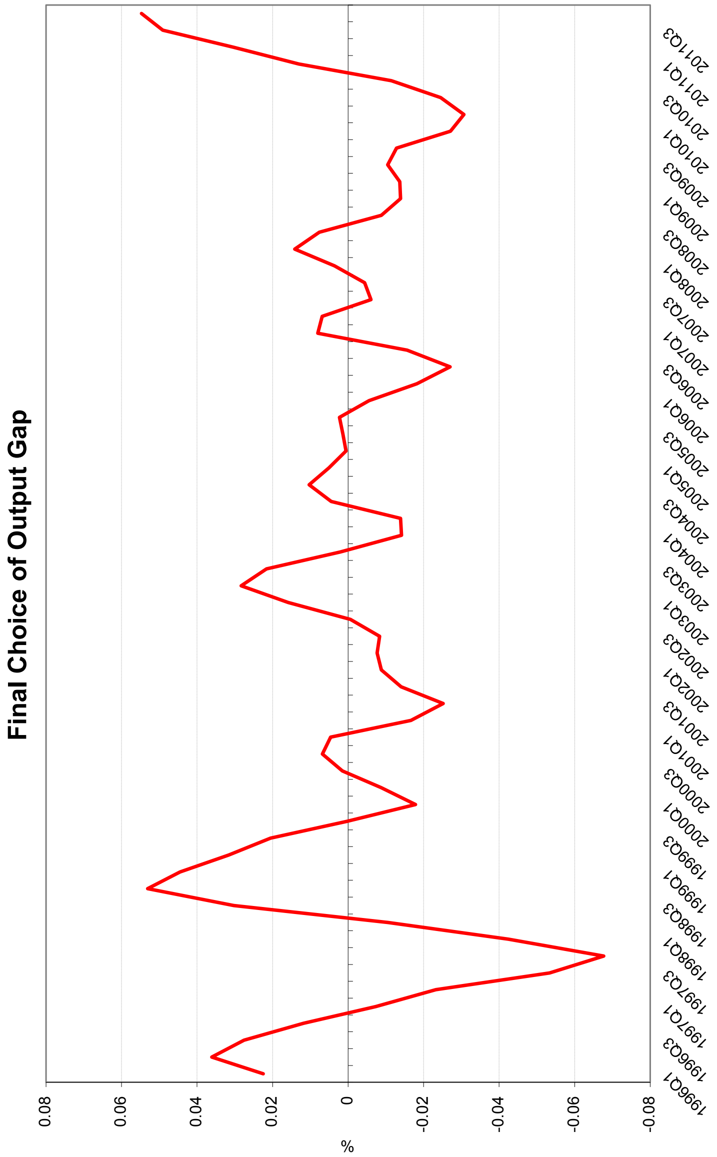

Figure 11: Final choice of output gap

Figure 12 shows these different measures.

4 Conclusion

In this paper, we constructed a quarterly measure of economic output using the procedure of
[Chow, Lin, 1971], employing a pre-filtering procedure to arrive at a sensible choice of quarterly indicator series, namely non-mineral employment and electricity production. We then employed a univariate HP filter to derive a quarterly measure of the output gap, and gave a brief interpretation of its implications for the historical profile of demand conditions in PNG over the last few decades.
This is far from the end of the story. As alluded to in the introduction, the true objective of constructing an output gap measure is to estimate a Phillip's Curve for PNG. This is a large project in its own right, and will form the basis for the second complimentary paper.
There are some clear issues that need to be resolved as well. For instance, the unaccountable movements in the output gap highlighted in Section 3.2 may be the result of inaccuracies in data, or a lack of economic understanding. In the first case, this is easily resolved through further data treatment. In the second case, this may be resolved through estimation of a Phillips Curve, as above, i.e. interpreting the output gap with respect to inflation and other supply-side variables could yield a more refined understanding of overall demand conditions during those periods.
This aside, there are still several avenues of further work available for the present paper. Firstly, an even greater set of potential indicator series could be utilized. For instance, the survey data from BPNG's Business Liaison Survey could be utilized, once it is fully verified and stripped of inaccuracies. Such a data set could be invaluable for further understanding drivers of quarterly output. Finally, using state-space methods to estimate the output gap through an UC approach could be a worthwhile endeavor.

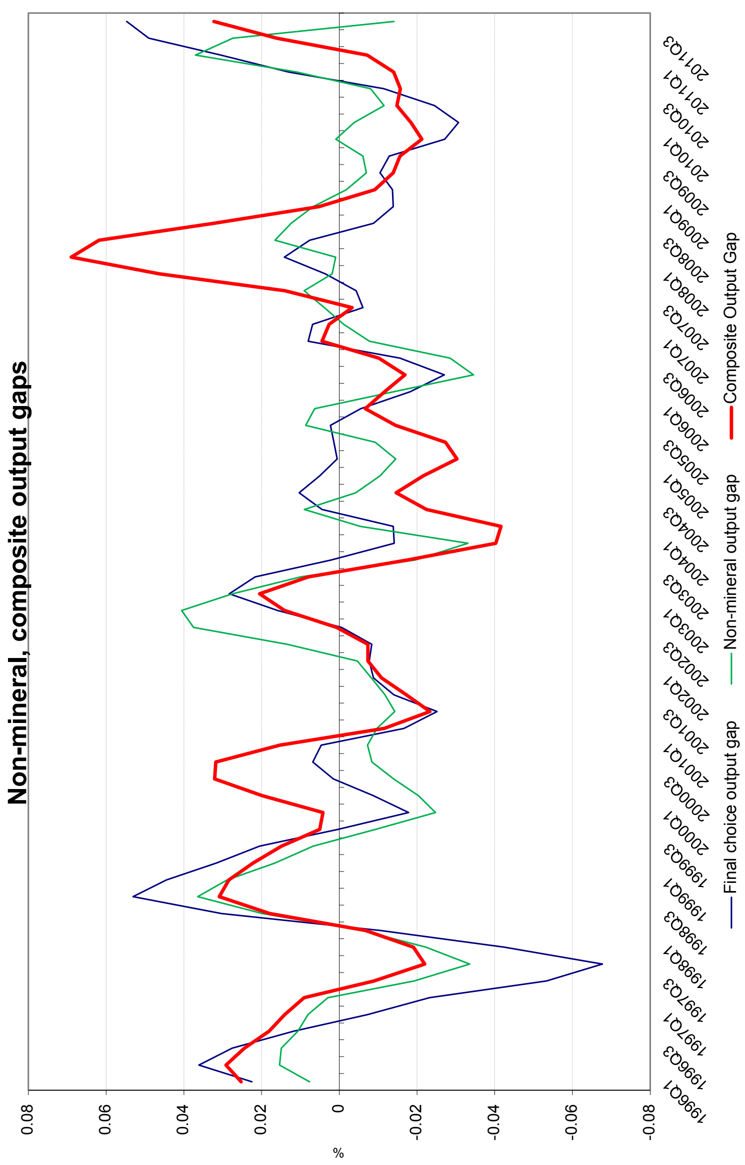

Figure 12: Final choice, non-mineral and composite output gaps

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A Description of Data

- **Gross Domestic Product**
  - Nominal - annual data, in Kina terms, 1996-2006 inclusive, provided by the National Statistics Office (NSO) of PNG. 2007-2011, based on Ministry of Treasury of PNG estimates.
  - Real - as for nominal GDP, in Kina terms, base years 1983 and 1998.
  - Deflator - as for nominal GDP.
  - Fitted Deflator - quarterly data, constructed by applying Chow-Lin procedure to the annual deflator, using headline CPI as a quarterly indicator.
- **Production data**
  - Electricity Production - quarterly data, in kilowatt-hours (kwh), 1996Q1 - 2011Q4, provided by PNG Power.
  - Employment Indices - quarterly data, index, 1996Q1 - 2011Q4, provided by BPNG.
- **Monetary data**
  - Nominal, all series - quarterly data, in Kina terms, 2002Q1 - 2011Q4, provided by BPNG.
  - Real - as for nominal, deflated using fitted GDP deflator.
- **Fiscal data**
  - Nominal, all series - quarterly data, in Kina terms, 1996Q1 - 2011Q4, provided by the Ministry of Treasury.
  - Real - as for nominal, deflated using fitted GDP deflator.
- **Imports data**
  - Nominal, all series - quarterly data, in Kina terms, 1996Q1 - 2011Q4, provided by BPNG.
  - Real - as for nominal, deflated using the International Monetary Fund (IMF) World Economic Outlook's (WEO) *food and beverage* and *energy* commodity price indices.

B Structure of the Covariance matrix

Under heteroskedasticity and pseudo-autocorrelation, the covariance matrix Ω is defined by:

$$\Omega = WVW = \frac{1}{1-\rho^2}\begin{pmatrix} \omega_1 & 0 & \cdots & 0 \\ 0 & \omega_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_{4n} \end{pmatrix} \times \begin{pmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{4n-1} \\ \rho & 1 & \rho & \cdots & \rho^{4n-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{4n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{4n-1} & \rho^{4n-2} & \rho^{4n-3} & \cdots & 1 \end{pmatrix}$$
$$\times \begin{pmatrix} \omega_1 & 0 & \cdots & 0 \\ 0 & \omega_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_{4n} \end{pmatrix}$$

In this case, if y_a is multivariate normal, then is has mean $CX\beta$ and variance $CW'VWC'$. As was noted earlier, we make the working assumption that the innovations are homoskedastic, i.e. that $W = I_{4n}$. In practice, a canonical choice for the $\omega_i's$ could be the associated weighting as determined through the PCA performed earlier (see [Frain, 2004]). The log-likelihood function is given by

$$\begin{aligned}\mathcal{L}(\beta|X) = -\frac{N}{2}log(2\pi) \quad - \quad & \frac{1}{2}log(|CW'VWC'|) \\ & -\frac{1}{2}(y_a - CX\beta)'(X_a'(CW'VWC')^{-1}X_a)^{-1}(y_a - CX\beta)\end{aligned}$$

For a given ρ, the solution to the maximum likelihood problem is given by:

$$\beta_{max} = (X_a'(CW'VWC')^{-1}X_a)^{-1}X_a'(CW'VWC')^{-1}y_a$$

and

$$\sigma^2_{max} = \frac{1}{N}(y_a - CX\beta_{max})'(CW'VWC')(y_a - CX\beta_{max})$$

Hence

$$\mathcal{L}(\beta_{max}) = -\frac{N}{2}(log(2\pi)+1) - \frac{N}{2}log(\sigma^2_{max}) - \frac{1}{2}log(|C'\Omega C|)$$

Thus, our task reduces to solving

$$\max_{\rho \in (-1,1)} \mathcal{L}(\beta_{max}(\rho))$$

In our routine, we achieve this through a relatively crude iterative procedure that determines the local maximum of $\mathcal{L}(\beta_{max})$ corresponding to the largest value of ρ. More specifically, if
$\{\mathcal{L}(\beta_{max}(\rho_1)), ..., \mathcal{L}(\beta_{max}(\rho_n))\}$ are the set of local maxima for $\mathcal{L}(\beta_{max}(\rho))$, corresponding to the values $\{\rho_1, ..., \rho_n\}$, then we pick ρ_k, where $\rho_k = max\{\rho_1, ..., \rho_n\}$.[25]
For illustrative purposes, Figure 13 plots $\mathcal{L}(\beta_{max}(\rho))$ for ρ in the interval $[-0.9, 0.9]$ for a few combinations of quarterly series:
The reason for this choice is that higher values of ρ generate smoother quarterly profiles, which in turn generate more sensible output gaps and seem to fit better with an intuitive understanding of the data. Figure 14 shows various quarterly output series based on fixed values of ρ.[26]

C Application of Univariate Filters

C.1 The HP Filter

The HP filter solves the constrained minimization problem:

$$\begin{aligned} \text{Minimize} \qquad & \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=1}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \\ \text{subject to } y_t \quad = \quad & \tau + c_t \end{aligned}$$

The constraint represents output as the sum of a trend component, τ_t, and a cyclical component, c_t. The minimization expression is comprised of two terms. The first term minimizes the magnitude of the cyclical component, whereas the second term penalizes variations in the growth rate of the trend component, i.e. smoothens the trend component. Thus, the parameter λ controls the smoothness of the trend component. The larger the value of λ, the smoother the trend component. [Hodrick, Prescott, 1980] suggest a value for λ of 1600, based on

[25] The routine actually finds local minima to the negative of the expression $\mathcal{L}(\beta_{max}(\rho))$.
[26] The quarterly series used for this graph are non-mineral employment, electricity production, mineral employment and currency in circulation.

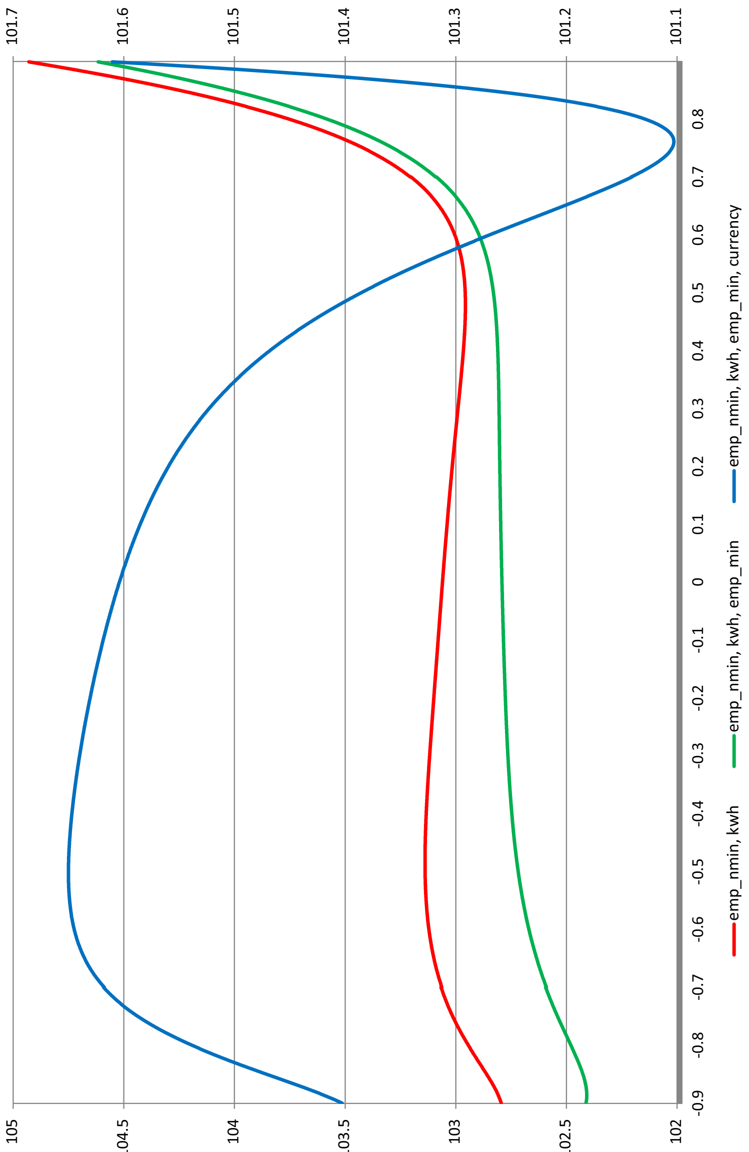

Figure 13: $\mathcal{L}(\beta_{max}(\rho))$ for different combinations of quarterly series

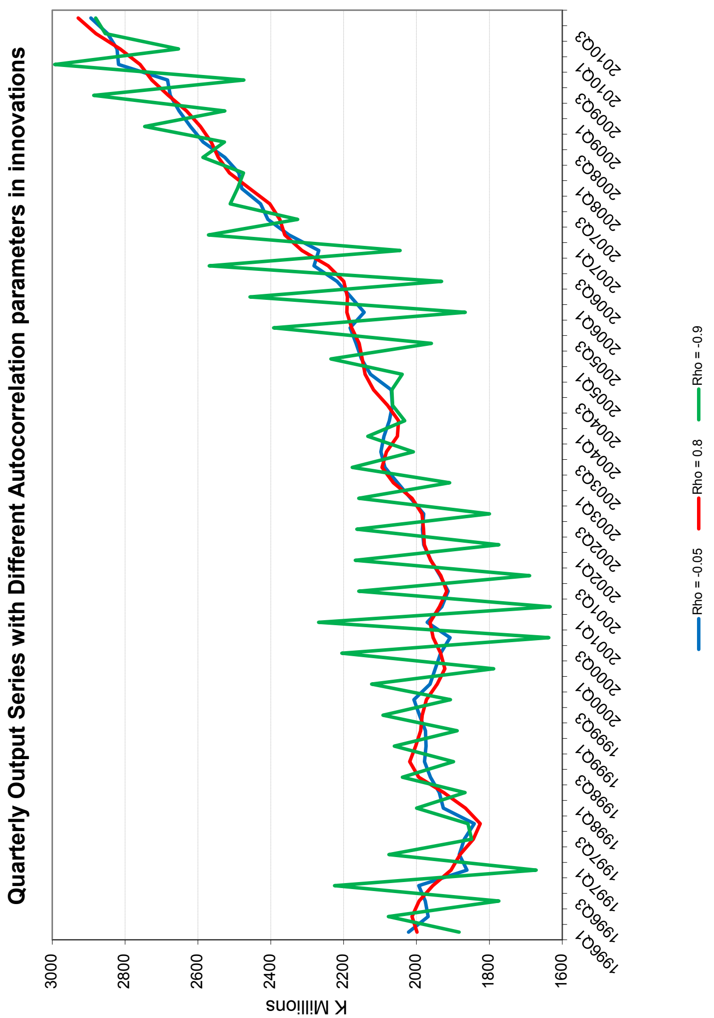

Figure 14: Quarterly Output Series with Different Autocorrelation parameters in innovations

US data. However, this choice is entirely a function of the authors' priors on the nature of potential output in the US over the prevailing period. As such, [Marcet, Ravn., 2003] suggest two "adjustment rules". The first keeps *relative* the volatility of growth in the trend and cycle components constant, the second keeps the *absolute* volatility of growth in the trend component constant. We wrote a simple program in Eviews that performs both adjustment rules. Applying adjustment rules 1 and 2 to the HP filter, based on an output series generated using non-mineral employment and electricity production as quarterly input series, yielded λ-values of 2290 and 3490 respectively. These values imply a slightly smoother profile for potential output, i.e. potential output in PNG is slightly slower to adjust. Figure 15 shows the consequences for the resultant output gaps.

The newly determined values for λ clearly have little effect on the overall profile of the output gap. As such, we will revert to using the default value of 1600.

C.2 The BN Decomposition

The BN decomposition yields a trend component τ_t based on the following expression:

$$\tau_t = \lim_{m \to \infty} E[y_{t+m} - m\mu | \Omega_t]$$

where $\mu = E[\Delta y_t]$ is the deterministic drift and Ω_t is the information set used to generate the conditional expectation.[27] Typically, the auto-covariance structure of the drift Δy_t is modeled as an auto-regressive moving-average (ARMA) process. Figure 16 shows a comparison of output gaps, using a variety of ARMA specifications. Note that series BN_p_q refers to a ARMA(p,q) specification. Whilst all the output gaps shown appear to demonstrate mild non-stationarity, the ARMA(1,1), ARMA(2,1) and ARMA(1,2) seem to exhibit the most credible profile. We will use the ARMA(2,1) henceforth.

[27] See [Morley., 2010].

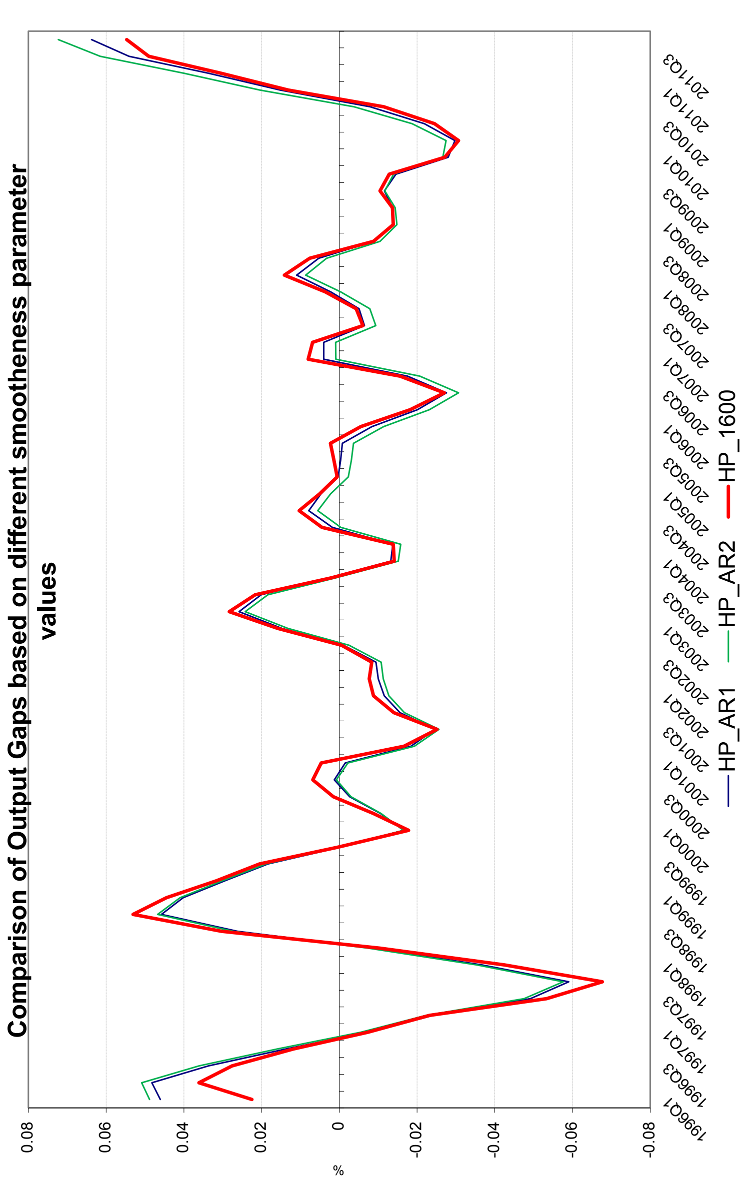

Figure 15: Comparison of output gaps based on different λ-values in the HP filter

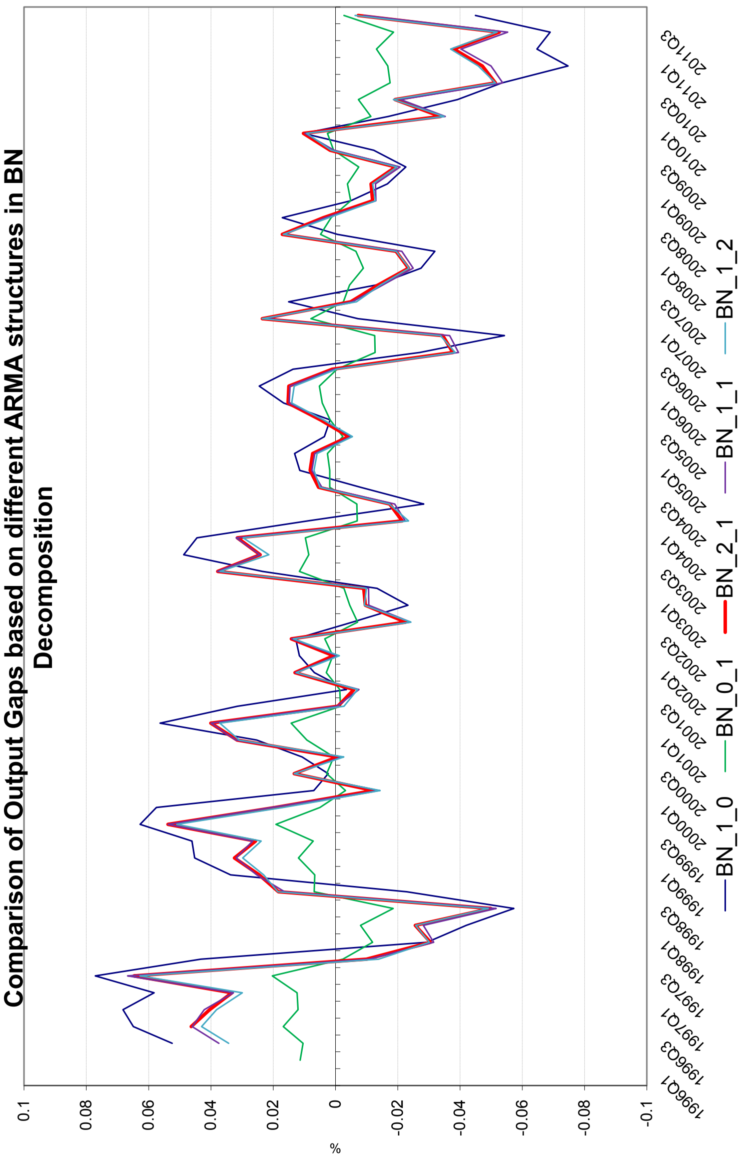

Figure 16: Comparison of output gaps based on different λ-values in the HP filter

D Eviews code for Chow-Lin procedure

```
'written by N.Vellodi, Research Department, Bank of Papua New Guinea

cd "C:\Documents\BPNG\Output Gap\Eviews PNG"
load gdp_indicators

smpl 1996:1 2010:4

'the low-frequency series to be disaggregated

genr y_lf_ser = gdp_cnst
genr y_hf_ser = gdp_cnst_qrtly

'the quarterly indicator series

genr x_hf_ser_1 = emp_nonmin_tc
genr x_hf_ser_2 = kwh_tc
genr x_hf_ser_3 = emp_min_tc
genr x_hf_ser_4 = currency_tc
genr x_hf_ser_5 = tax_per_tc
genr x_hf_ser_6 = exp_curr_tc

'NOTE: you may include arbitrarily more indicator series.

scalar s = 4    'the frequency of disaggregation, s=4 annual to
    quarterly, s=12 annual to monthly, s=3 quarterly to monthly


scalar pi = 3.14159265
scalar N_ind = 4                'the number of indicator series
    being used
scalar N_q = 60         'the number of quarterly observations; must
    match the sample period defined above
scalar N_y = 15         'the number of annual observations



'==============================================

'subroutine to create aggregation matrix C.
```

```
subroutine local aggreg (matrix AGG, scalar op1, scalar N, scalar s,
     scalar n_up, scalar n_dn)

'________________________________________________
'        PURPOSE:          Generate a temporal aggregation matrix
'________________________________________________
'        SYNTAX:           matrix AGG
'                                          call aggreg (AGG, op1, N, s,
     n_up, n_dn)
'________________________________________________
'        OUTPUT:           N x sN temporal aggregation matrix AGG(NxsN)
'________________________________________________
'        INPUT:            AGG:            aggregation matrix name,
    have to be declared before subroutine
'                                          op1:            type of
    temporal aggregation
'                                          op1=1           sum (flow)
'                                          op1=2           average (
    index)
'                                          op1=3   last   element (stock
    ) - interpolation
'                                          op1=4   first  element (
    stock) - interpolation
'                                          N:
    number of low frequency data points
'                                          s:
    number of high frequency points
'                                                                    for
    each low frequency data points (freq. conversion)
'                                          s=4             annual to
    quarterly
'                                          s=12            annual to
    monthly
'                                          s=3             quarterly to
     monthly
'                                          n_up            number of
    extrapolated forward high frequency points
'                                                                    HF
    subperiods not subject to temporal aggreagation constraint
'                                          n_dn            number of
    extrapolated backward high frequency points
'                                                                    HF
    subperiods not subject to temporal aggreagation constraint

'________________________________________________
```

```
'        Checking parameter s

if (s=4 OR s=12 OR s=3) then

'        Generation of aggregation vector cc_vec

if op1=1 then
        rowvector(s) cc_vec = 1
else
        if op1=2 then
                rowvector(s) cc_vec = 1/s
        else
                if op1=3 then
                        rowvector(s) cc_vec = 0
                        cc_vec(s)=1
                else
                        if op1=4 then
                                rowvector(s) cc_vec = 0
                                cc_vec(1)=1
                        else
                        statusline  ERROR!!! AGG() subroutine !!!***
                             Improper value of option parameter [ta:
                             aggregation type = (1 to 4)] ***!!!
                        stop
                        endif
                endif
        endif
endif

'_____________________________________________

'        Generation of aggregation matrix AGG=I(N) kronecker cc_vec

if (n_up=0 AND n_dn=0) then

'        Generation of ordinary aggregation matrix, pure distribution
     n=N*s

matrix AGG=@kronecker(@identity(N), cc_vec)
else

'        Generation of enhaced aggregation matrix

        if (n_up>0 AND n_dn=0) then
```

```
            matrix AGG_temp=@kronecker(@identity(N), cc_vec)
            matrix(N, N*s+n_up) AGG
            matrix(N,  n_up) zeros
            matplace(AGG, AGG_temp,1,1)
            matplace(AGG, zeros, 1, N*s+1)
      else

      if (n_up=0 AND n_dn>0) then
            matrix AGG_temp=@kronecker(@identity(N), cc_vec)
            matrix(N, N*s+n_dn) AGG
            matrix(N,  n_dn) zeros
            matplace(AGG, AGG_temp,1,n_dn+1)
            matplace(AGG, zeros, 1, 1)
      else

      if (n_up>0 AND n_dn>0) then
            matrix AGG_temp=@kronecker(@identity(N), cc_vec)
            matrix(N, N*s+n_up+n_dn) AGG
            matrix(N,  n_dn) zeros_dn
            matrix(N,  n_dn) zeros_up
            matplace(AGG, AGG_temp,1,n_dn+1)
            matplace(AGG, zeros_dn, 1, 1)
            matplace(AGG, zeros_up, 1, N*s+1+n_dn)

      else
      statusline  ERROR!!! AGG() subroutine !!!*** Improper value
          of option parameter [n_up or n_dn: >= 0] ***!!!
      stop

endif
endif
endif
endif

else
statusline  ERROR!!! AGG() subroutine !!!*** Improper value of
    option parameter [s: frequency conv. = (3, 4, 12)] ***!!!
stop
endif


endsub

'=================================================
'=================================================
```

```
'============================================================

'         Asiggning series to vectors

sample NA_smpl_y @all if y_lf_ser <> NA
sample NA_smpl_x @all if x_hf_ser_1 <> NA

'         Assigning series to matrix objects

stomna(y_lf_ser, y_lf_temp)

matrix (N_q,N_ind) x_hf_temp

!i=1
while !i<=N_ind
stomna(x_hf_ser_{!i}, x_hf_vec_{!i})
colplace(x_hf_temp,x_hf_vec_{!i},!i)
!i = !i +1
wend

'____________________________________________________________

'         Size of the problem

!n_hf=@rows(y_lf_temp)
!n_lf=N_y
                        ' Size of low frequency input

!n_hf_x=@rows(x_hf_temp)                                        '
    Size of high frequency input (number of observations)
!p_hf_x_temp=@columns(x_hf_temp)                  ' Size of high
    frequency input (number of variables without intercept)
!p_hf_x=!p_hf_x_temp+1                                  ' Size of
    high frequency input (number of variables with intercept)

'____________________________________________________________
```

```
'        Aggregation of low frequency series equaly distributed in
    subperiods

matrix AGG1
call aggreg (AGG1, 1,!n_lf,s, 0, 0)
matrix(N_y,N_q) AGG2
!i=1
while (!i <= N_q)
        colplace(AGG2,@columnextract(AGG1,!i),!i)
        !i=!i+1
wend
matrix y_lf=AGG2*y_lf_temp
delete AGG1

'_______________________________________________

'        Generation of aggregation matrix AGG [!n_lf x !n_lf*s] for
    aggregation in procedure

matrix AGG
call aggreg (AGG, 1, !n_lf, s, 0, 0)

'_______________________________________________

'        Preparing the X matrix: including intercept

matrix(!n_hf_x, !p_hf_x) x_hf=1
matplace(x_hf, x_hf_temp , 1, 1)

'_______________________________________________

'        Temporal aggreagation of the indicator

matrix x_lf=AGG2*x_hf


'_______________________________________________
'subroutine calculating maximised value of log-likelihood function
    as a function of rho

subroutine f_rho (scalar frho, scalar Rho)
        matrix I=@identity(!n_hf_x)
        matrix W=I
        matrix(!n_hf_x, !n_hf_x) LL=0
        for !i=2 to !n_hf_x
```

```
        LL(!i, !i-1)=-1

                                          'Auxiliary matrix useful
              to simplify computations
    next
    scalar N = !n_lf
    matrix AUX=I+Rho*LL
    Aux(1,1)=@sqrt(1-Rho^2)
    matrix W=@inverse(@transpose(AUX)*AUX)

                         ' High frequency VCV matrix (without
        sigma_a)
    matrix WW=AGG2*W*@transpose(AGG2)'

                                    ' Low frequency VCV matrix (
        without sigma_a) EQUATION 4 IN CHOW LIN
    matrix Wi=@inverse(WW)
    matrix INV= @inverse(@transpose(x_lf)*Wi*x_lf)
    matrix beta= INV*@transpose(x_lf)*Wi*y_lf

                                    ' beta ML estimator EQUATION 15
        IN CHOW LIN
    matrix u_lf=y_lf-x_lf*beta


        ' Low frequency residuals EQUATION 16 IN CHOW LIN
    matrix scp=@transpose(u_lf)*Wi*u_lf

                                          ' Weighted least squere
    matrix sigma_a=scp*(1/N)

                                                            '

        sigma_a ML estimator
    matrix FM=w*@transpose(AGG2)*Wi

                                    ' Filtering matrix EQUATION 14 (
        Vz.V.^(-1))
    matrix u_hf=FM*u_lf
    scalar scp_scalar = scp(1,1)
    scalar sigma_scalar = sigma_a(1,1)
    scalar frho = N/2*(log(2*pi)) + N/2*log(sigma_scalar) + 1/2*
        log(@det(WW)) + 1/(2*sigma_scalar)*(scp_scalar)
endsub
```

```
'_______________________________________________
'subroutine calculating value of rho that maximises log-likelihood
    function using a manual search. Specifically, it finds the
    highest local maximum of the likelihood function.

subroutine rhomaxvector (matrix frhos)

        !k_t = 1800
        !k_step = 0.001
        !k_min = -0.9
        !frhos_dim = !k_t
        matrix(!k_t,1) frhos = 0
        !k=1
        while !k<= !k_t
        !frho_k = 0
        call f_rho(!frho_k, !k_min+!k*!k_step)
        frhos(!k,1)=!frho_k
        !k = !k +1
        wend

endsub

subroutine rhomaxmanual (scalar rho2)
        matrix frhos1 = 0
        call rhomaxvector(frhos1)
        !k = 1798
        while (frhos1(!k+2,1) - frhos1(!k+1,1)>0)
                        if (frhos1(!k,1) - frhos1(!k+1,1)>0) then
                                rho2 =  -0.9 + 0.001*(!k+1)
                        endif
        !k=!k-1
        wend

endsub

'_______________________________________________
'_______________________________________________
'       Final estimation with fixed rho

subroutine final (series qrtly_indicator)

matrix I=@identity(!n_hf_x)
matrix W=I
matrix(!n_hf_x, !n_hf_x) LL=0
```

```
for !i=2 to !n_hf_x
        LL(!i, !i-1)=-1
next
        scalar rho1 = 0
        call rhomaxmanual(rho1)
        if(rho1<-0.8) then
                statusline  ERROR!!! final() subroutine !!!*** BAD
                    RHO ***!!!
        stop
        endif
        scalar N = N_y
'       scalar rho3 = -0.9
        matrix AUX=I+rho1*LL
'       matrix AUX=I+rho3*LL
        Aux(1,1)=@sqrt(1-rho1^2)
'       Aux(1,1)=@sqrt(1-rho3^2)
        matrix W=@inverse(@transpose(AUX)*AUX)

                    ' High frequency VCV matrix (without sigma_a)
        matrix WW=AGG2*W*@transpose(AGG2)

                            ' Low frequency VCV matrix (without
            sigma_a) EQUATION 4 IN CHOW LIN
        matrix Wi=@inverse(WW)
        matrix beta=@inverse(@transpose(x_lf)*Wi*x_lf)*@transpose(
            x_lf)*Wi*y_lf ' beta ML estimator EQUATION 15 IN CHOW
            LIN
        matrix u_lf=y_lf-x_lf*beta

                                                                   ' Low
            frequency residuals EQUATION 16
        matrix scp=@transpose(u_lf)*Wi*u_lf

                                   ' Weighted least squere
        matrix sigma_a=scp*(1/N)

                                                          ' sigma_a ML
            estimator
        matrix FM=w*@transpose(AGG2)*Wi

                            ' Filtering matrix EQUATION 14 (Vz.V
            .^(-1))
        matrix u_hf=FM*u_lf

                                                                   ' High
```

```
             frequency residuals

'_________________________________________

'_________________________________________

'        Temporaly disaggregated time series

matrix y_hf=x_hf*beta+u_hf



'        Assigning vector to series

mtos(y_hf, y_hf_ser, NA_smpl_x)
series qrtly_output = y_hf_ser

endsub

series qrtly_output
call final (qrtly_output)
genr gdp_annual = gdp_cnst
line qrtly_output gdp_annual
```